



## Higher derivations associated with the Cauchy–Jensen type mapping

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### Abstract

Let  $H$  be an infinite–dimensional Hilbert space and  $K(H)$  be the set of all compact operators on  $H$ . We will adopt spectral theorem for compact self-adjoint operators, to investigate of higher derivation and higher Jordan derivation on  $K(H)$  associated with the following cauchy–Jensen type functional equation

$$2f\left(\frac{T+S}{2} + R\right) = f(T) + f(S) + 2f(R)$$

for all  $T, S, R \in K(H)$ .

*Key words:* Cauchy–Jensen type higher derivation, Cauchy–Jensen type higher Jordan derivation, Approximate–strong,  $C^*$ –algebra.

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## 1 Introduction

If  $H$  is a Hilbert space, then an operator  $T$  in  $B(H)$  is called a compact operator if the image of unit ball of  $H$  under  $T$  is a compact subset of  $H$ . Note that if the operator  $T : H \rightarrow H$  is compact, then the adjoint of  $T$  is compact, too. The set of all compact operators on  $H$  is shown by  $K(H)$ . It is easy to see that  $K(H)$  is a  $C^*$ -algebra [3]. Moreover, every operator on  $H$  with finite range is compact. We denote by  $P(H)$  the set of all finite range projections on Hilbert space  $H$ .

An approximate unit for a  $C^*$ -algebra  $\mathcal{A}$  is an increasing net  $(u_\lambda)_{\lambda \in \Lambda}$  of positive elements in the closed unit ball of  $\mathcal{A}$  such that  $a = \lim_\lambda au_\lambda = \lim_\lambda u_\lambda a$  for all  $a \in \mathcal{A}$ . Every  $C^*$ -algebra admits an approximate unit [4].

**Example 1.1** *Let  $H$  be a Hilbert space with orthonormal basis  $(e_n)_{n=1}^\infty$ . The  $C^*$ -algebra  $K(H)$  is non-unital, since  $\dim(H) = \infty$ . If  $P_n$  is a projection on  $\mathbb{C}e_1 + \dots + \mathbb{C}e_n$ , then the increasing sequence  $(P_n)_{n=1}^\infty$  is an approximate unit for  $K(H)$ .*

**Theorem 1.1** ([4]). *Let  $T : H \rightarrow H$  be a compact self-adjoint operator on Hilbert space  $H$ . Then there is an orthonormal basis of  $H$  consisting of eigenvectors of  $T$ . The nonzero eigenvalues of  $T$  are from finite or countably infinite set  $\{\lambda_k\}_{k=1}^\infty$  of real numbers and  $T = \sum_{k=1}^\infty \lambda_k P_k$ , where  $P_k$  is the orthogonal projection on the finite-dimensional space of eigenvectors corresponding to eigenvalues. If the number of nonzero eigenvalues is countably infinite, then the series converges to  $T$  in the operator norm.*

The problem of stability of functional equations originated from a question of Ulam [5] concerning the stability of group homomorphisms: let  $(G_1, *)$  be a group and let  $(G_2, \star, d)$  be a metric group with the metric  $d(., .)$ . Given  $\varepsilon > 0$ , does there exist a  $\delta(\varepsilon) > 0$  such that if a mapping  $h : G_1 \rightarrow G_2$  satisfies the inequality

$$d(h(x * y), h(x) \star h(y)) < \delta$$

for all  $x, y \in G_1$ , then there exists a homomorphism  $H : G_1 \rightarrow G_2$  with

$$d(h(x), H(x)) < \varepsilon$$

for all  $x \in G_1$ ? If the answer is affirmative, we would say that the equation of homomorphism  $H(x * y) = H(x) * H(y)$  is stable. Thus, the stability question of functional equations is that how do the solutions of the inequality differ from those of the given functional equation?

Hyers [6] gave the first affirmative answer to the question of Ulam for Banach spaces. Let  $X$  and  $Y$  be Banach spaces. Assume that  $f : X \rightarrow Y$  satisfies

$$\|f(x + y) - f(x) - f(y)\| \leq \varepsilon$$

for all  $x, y \in X$  and some  $\varepsilon > 0$ . Then, there exists a unique additive mapping  $T : X \rightarrow Y$  such that

$$\|f(x) - T(x)\| \leq \varepsilon$$

for all  $x \in X$ . This method is called the direct method or Hyers-Ulam stability of functional equations.

Let  $\mathbb{N}$  be the set of natural numbers. For  $m \in \mathbb{N} \cup \{0\} = \mathbb{N}_0$ , a sequence  $H = \{h_0, h_1, \dots, h_m\}$  (resp.  $H = \{h_0, h_1, \dots, h_n, \dots\}$ ) of linear mappings from  $C^*$ -algebra  $A$  into  $C^*$ -algebra  $B$  is called a higher derivation of rank  $m$  (resp. infinite rank) from  $A$  into  $B$  if

$$h_n(xy) = \sum_{l+j=n} h_l(x)h_j(y)$$

holds for each  $n \in \{0, 1, \dots, m\}$  (resp.  $n \in \mathbb{N}_0$ ) and all  $x, y \in A$ . A higher derivation  $H$  from  $A$  into  $B$  is said to be continuous if each  $h_n$  is continuous on  $A$ . The higher derivation  $H$  on  $A$  is called be strong if  $h_0$  is an identity mapping on  $A$ . Of course, a higher derivation of rank 0 from  $A$  into  $B$  (resp. a strong higher derivation of rank 1 on  $A$ ) is a homomorphism (resp. a derivation). So a higher derivation is a generalization of both a homomorphism and a derivation.

**Definition 1.1** *Let  $\mathcal{A}$  be a  $C^*$ -algebra without unit. A sequence  $H = \{h_0, h_1, \dots, h_m, \dots\}$  of mapping from  $\mathcal{A}$  into  $\mathcal{A}$  is called approximate -*

strong if,  $\lim_n h_0(x_n)x = x \lim_n h_0(x_n) = x$  for all  $x \in \mathcal{A}$  when  $\{x_n\}_n$  is approximate unit in  $\mathcal{A}$ .

**Theorem 1.2** [16] *Let  $X$  is a normed spaces and  $Y$  is a Banach space. If  $f : X \longrightarrow Y$  be mapping for which there exists a function  $\psi : X^3 \longrightarrow [0, \infty)$  such that;*

$$\sum_j \frac{1}{2^j} \psi(2^j x, 2^j y, 2^j z) < \infty$$

and

$$\|2f\left(\frac{x+y}{2} + z\right) - f(x) - f(y) - 2f(z)\| < \psi(x, y, z)$$

for all  $x, y, z \in X$ . Then, there exists a unique additive mapping  $L : X \longrightarrow Y$  such that

$$\|f(x) - L(x)\| < \frac{1}{4} \psi(x, x, x)$$

for all  $x \in X$ .

In this paper, we will adopt spectral theorem for compact self-adjoint operators, to investigate of higher derivation and higher Jordan derivation on  $K(H)$  associated with the following cauchy–Jencen type functional equation

$$2f\left(\frac{T+S}{2} + R\right) = f(T) + f(S) + 2f(R)$$

for all  $T, S, R \in K(H)$ .

## 2 Higher derivation on $K(H)$

It is easy to see that if a continuous mapping  $f : X \longrightarrow Y$  with  $f(ix) = if(x)$  for all  $x \in X$  satisfy conditions of theorem 1.2, then the mapping  $L : X \longrightarrow Y$  given in statement of theorem 1.2 is a  $\mathbb{C}$ –linear. We us this fact in this paper.

**Lemma 2.1** *Assume that a mapping  $f : X \longrightarrow B$  is additive and for*

each fixed  $x \in X$ ,  $f(\lambda x) = \lambda f(x)$  for all  $\lambda \in \mathbb{T}_{\theta_0}^1 := \{e^{i\theta} : 0 \leq \theta \leq \theta_0\}$ . Then  $f$  is  $\mathbb{C}$ -linear.

**Proof.** If  $\lambda$  belong to  $\mathbb{T}^1$ , then there exists  $\theta \in [0, 2\pi]$  such that  $\lambda = e^{i\theta}$ . It follows from  $\frac{\theta}{n} \rightarrow 0$  as  $n \rightarrow \infty$  there exists  $n_0 \in \mathbb{N}$  such that  $\lambda_1 = e^{i\frac{\theta}{n}}$  belong to  $\mathbb{T}_{\theta_0}^1$  and  $f(\lambda x) = f(\lambda_1^{n_0} x) = \lambda_1^{n_0} f(x) = \lambda f(x)$  for all  $x \in X$ . Let  $t \in (0, 1)$ . putting  $t_1 = t + i(1 - t^2)^{\frac{1}{2}}$ ,  $t_2 = t - i(1 - t^2)^{\frac{1}{2}}$ . Then we have  $t = \frac{t_1 + t_2}{2}$  and  $t_1, t_2 \in \mathbb{T}^1$ . It follows that

$$f(tx) = f\left(\frac{t_1 + t_2}{2}x\right) = \frac{t_1}{2}f(x) + \frac{t_2}{2}f(x) = tf(x).$$

If  $\lambda \in B_1 := \{\lambda \in \mathbb{C}; |\lambda| \leq 1\}$ , then there exists  $\theta \in [0, 2\pi]$  such that  $\lambda = |\lambda| e^{i\theta}$ . It follows that

$$f(\lambda x) = f(|\lambda| e^{i\theta} x) = |\lambda| f(e^{i\theta} x) = \lambda f(x).$$

for all  $x \in X$ . If  $\lambda \in \mathbb{C}$  then, there exist  $n_0 \in \mathbb{N}$  (from  $\frac{\lambda}{n} \rightarrow 0$  as  $n \rightarrow \infty$ ) such that  $\lambda_0 = \frac{\lambda}{n_0} \in B_1$ . It follows that

$$f(\lambda x) = f(n_0 \lambda_0 x) = n_0 \lambda_0 f(x) = \lambda f(x)$$

for all  $x \in X$ .  $\square$

**Lemma 2.2** Let  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is a sequence of continuous mappings from  $K(H)$  into  $K(H)$  such that;  $\varphi_m(TP) = \sum_{l+j=m} \varphi_l(P)\varphi_j(T)$  for all  $T \in K(H)$  and  $P \in P(H)$ .

- 1) If  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is an approximate-strong, then for approximate unit  $\{P_n\} \subset P(H)$ , we have  $\lim_n \varphi_m(P_n) = 0$  for each  $m \in \{1, 2, 3, \dots\}$ .
- 2) If  $\varphi_0(0) = 0$ , then  $\varphi_m(0) = 0$  for each  $m \in \{1, 2, 3, \dots\}$ .

**Proof.** It is clear.  $\square$

**Definition 2.1** Let  $\mathcal{A}$  be a  $C^*$ -algebra. A sequence  $H = \{h_0, h_1, \dots, h_m, \dots\}$  of mapping from  $\mathcal{A}$  into  $\mathcal{A}$  with  $h_0(0) = 0$  is called a Cauchy-Jensen type

higher derivation if for each  $m \in \mathbb{N}_0$

$$h_m(xy) = \sum_{l+j=m} h_l(x)h_j(y)$$

and

$$2h_m\left(\frac{x+y}{2} + 2\lambda z\right) = h_m(x) + h_m(y) + 2\lambda h_m(z)$$

for all  $x, y, z \in \mathcal{A}$  and  $\lambda \in \mathbb{C}$ .

**Theorem 2.1** *Let  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is an approximate-strong of continuous mappings from  $K(H)$  into  $K(H)$  for which for each  $m \in \mathbb{N}_0$  there exists a function  $\psi_m : K(H)^3 \rightarrow [0, \infty)$  such that;*

$$\sum_j \frac{1}{2^j} \psi_m(2^j T, 2^j S, 2^j R) < \infty \quad (2.1)$$

and

$$\|2\varphi_m\left(\frac{T+S}{2} + \lambda R\right) - \varphi_m(T) - \varphi_m(S) - 2\lambda\varphi_m(R)\| < \psi_m(T, S, R) \quad (2.2)$$

for all  $T, S, R \in K(H)$ ,  $\lambda \in \{1, i\}$  and  $m \in \mathbb{N}_0$ .

If  $\varphi_m(2^n T P) = \sum_{l+j=m} 2^n \varphi_l(T) \varphi_j(P)$  for each  $T \in K(H)$  and  $P \in P(H)$ , then  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is a cauchy-Jensen type higher derivation.

**Proof.** From continuity of  $\varphi_m$  and by the same reasoning as in the proof of the theorems of [16], for each  $m \in \mathbb{N}_0$ , there exists  $\mathbb{R}$ -linear mapping  $h_m : K(H) \rightarrow K(H)$  with  $h_m(T) = \lim_{n \rightarrow \infty} \frac{1}{2^n} \varphi_m(2^n T)$  such that

$$\|h_m(T) - \varphi_m(T)\| < \frac{1}{4} \psi_m(T, T, T)$$

for all  $T \in K(H)$ . It follows from (2.2) and lema 2.1 that  $h_m$  is a  $\mathbb{C}$ -linear for each  $m \in \mathbb{N}_0$ .

We show that  $h_m \equiv \varphi_m$  for each  $m \in \mathbb{N}_0$ . Let  $\{P_k\} \subset P(H)$  be approxi-

mate unit of  $K(H)$ . Then by lemma 2.2 and linearity of  $\varphi_m$  we get

$$\begin{aligned}
h_m(T) &= \lim_n \frac{1}{2^n} \varphi_m(2^n T) = \lim_{n,k} \frac{1}{2^n} \varphi_m(2^n T P_k) \\
&= \lim_{n,k} \frac{1}{2^n} \sum_{l+j=m} 2^n \varphi_l(P_k) \varphi_j(T) \\
&= \lim_k \sum_{l+j=m} \varphi_l(P_k) \varphi_j(T) = \varphi_m(T)
\end{aligned}$$

for all  $T \in K(H)$  and  $m \in \mathbb{N}_0$ . Now, let  $S, T \in K(H)$ . There are compact self adjoint operators  $S_1, S_2$  such that  $S = S_1 + iS_2$ . According to Theorem 1.1 we have  $S = S_1 + iS_2 = \sum_{l=1}^{\infty} \alpha_l P_l + i \sum_{j=1}^{\infty} \beta_j P_j$  where  $P_k \in P(H)$  and  $\alpha_k, \beta_k \in \mathbb{C}$  for all  $k \in \{1, 2, 3, \dots\}$ . It follows from linearity and continuity of  $\varphi$  and  $T$  that

$$\begin{aligned}
\varphi_m(TS) &= \varphi_m\left(T\left\{\sum_{l=1}^{\infty} \alpha_l P_l + i \sum_{j=1}^{\infty} \beta_j P_j\right\}\right) \\
&= \sum_{l=1}^{\infty} \varphi_m(T\alpha_l P_l) + i \sum_{j=1}^{\infty} \varphi_m(T\beta_j P_j) \\
&= \sum_{l=1}^{\infty} \sum_{s+k=m} \varphi_k(T) \varphi_s(\alpha_l P_l) + i \sum_{j=1}^{\infty} \sum_{s+k=m} \varphi_k(T) \varphi_s(\beta_j P_j) \\
&= \sum_{s+k=m} \varphi_k(T) \sum_{l=1}^{\infty} \varphi_s(\alpha_l P_l) + i \sum_{s+k=m} \varphi_k(T) \sum_{j=1}^{\infty} \varphi_s(\beta_j P_j) \\
&= \sum_{s+k=m} \varphi_k(T) \left\{ \sum_{l=1}^{\infty} \varphi_s(\alpha_l P_l) + i \sum_{j=1}^{\infty} \varphi_s(\beta_j P_j) \right\} \\
&= \sum_{k+s=m} \varphi_k(T) \varphi_s(S).
\end{aligned}$$

This means that  $\phi$  is a cauchy–Jensen type higher derivation.  $\square$

**Corollary 1** *Let  $p \in (0, 1)$ ,  $\theta \in [0, \infty)$  be real numbers. Suppose that  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is an approximate–strong of continuous mappings from  $K(H)$  into  $K(H)$  with  $\varphi_0(0) = 0$  such that,  $\varphi_m(2^n TP) = \sum_{l+j=m} 2^n \varphi_l(T) \varphi_j(P)$  for each  $m \in \mathbb{N}_0$ ,  $T \in K(H)$  and  $P \in P(H)$ .*

If

$$\|2\varphi_m\left(\frac{T+S}{2} + \lambda R\right) - \varphi_m(T) - \varphi_m(S) - 2\lambda\varphi_m(R)\| < \theta(\|T\|^p + \|S\|^p + \|S\|^p)$$

for all  $\lambda \in \{1, i\}$  and all  $T, S, R \in K(H)$  and for each  $m \in \mathbb{N}_0$ , then  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is a Cauchy–Jensen type higher derivation.

**Proof.** Setting  $\phi(T, S, R) := \theta(\|T\|^p + \|S\|^p + \|R\|^p)$  all  $T, S, R \in K(H)$ . Then by Theorem 2.1 we get the desired result.  $\square$

**Corollary 2** Let  $p_1, p_2, p_3$  and  $\theta$  be positive real numbers with  $p_1 + p_2 + p_3 < 1$  and let  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  be an approximate–strong of continuous mappings from  $K(H)$  into  $K(H)$  with  $\varphi_0(0) = 0$  such that,  $\varphi_m(2^n TP) = \sum_{l+j=m} 2^n \varphi_l(T) \varphi_j(P)$  for each  $m \in \mathbb{N}_0$ ,  $T \in K(H)$  and  $P \in P(H)$ .

If

$$\|2\varphi_m\left(\frac{T+S}{2} + \lambda R\right) - \varphi_m(T) - \varphi_m(S) - 2\lambda\varphi_m(R)\| < \theta(\|T\|_1^p \cdot \|S\|_2^p \cdot \|S\|_3^p)$$

for all  $\lambda \in \{1, i\}$  and all  $T, S, R \in K(H)$  and for each  $m \in \mathbb{N}_0$ , then  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is a Cauchy–Jensen type higher derivation.

**Proof.** Setting  $\phi(T, S, R) := \theta(\|T\|_1^p \cdot \|S\|_2^p \cdot \|R\|_3^p)$  all  $T, S, R \in K(H)$ . Then by Theorem 2.2 we get the desired result.  $\square$

### 3 Higher Jordan derivations on $K(H)$

**Definition 3.1** Let  $\mathcal{A}$  be a  $C^*$ –algebra. A sequence  $H = \{h_0, h_1, \dots, h_m, \dots\}$  of mapping from  $\mathcal{A}$  into  $\mathcal{A}$  with  $h_0(0) = 0$  is called a Cauchy–Jensen type higher Jordan derivation if for each  $m \in \mathbb{N}_0$ ,

$$h_m(xy) = \sum_{l+j=m} [h_l(x)h_j(y) + h_j(x)h_l(y)]$$



and

$$2h_m\left(\frac{x+y}{2} + \lambda z\right) = h_m(x) + h_m(y) + 2\lambda h_m(z)$$

for all  $x, y, z \in \mathcal{A}$  and  $\lambda \in \mathbb{C}$ .

**Theorem 3.1** Let  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is an approximate-strong of continuous mappings from  $K(H)$  into  $K(H)$  for which for each  $m \in \mathbb{N}_0$  there exists a function  $\psi_m : K(H)^3 \rightarrow [0, \infty)$  such that;

$$\sum_j \frac{1}{2^t} \psi_m(2^t T, 2^t S, 2^t R) < \infty$$

and

$$\|2\varphi_m\left(\frac{T+S}{2} + \lambda R\right) - \varphi_m(T) - \varphi_m(S) - 2\lambda\varphi_m(R)\| < \psi_m(T, S, R)$$

for all  $T, S, R \in K(H)$  and  $m \in \mathbb{N}_0$ . If  $\varphi_m(2^n TP + 2^n PT) = \sum_{l+j=m} [2^n \varphi_l(T)\varphi_j(P) + 2^n \varphi_j(P)\varphi_l(T)]$  for all  $T \in K(H)$  and  $P \in P(H)$ , then  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is a higher Jordan derivation.

**Proof.** By the same reasoning as the proof of Theorem 2.1, for each  $m \in \mathbb{N}_0$ , there exists a unique  $\mathbb{C}$ -linear mapping  $h_m : K(H) \rightarrow K(H)$  with  $h_m(T) = \lim_{n \rightarrow \infty} \frac{1}{2^n} \varphi(2^n T)$  such that

$$\|h_m(T) - \varphi_m(T)\| < \frac{1}{4} \psi(T, T, T)$$

for all  $T \in K(H)$  and  $m \in \mathbb{N}_0$ .

We show that  $h_m \equiv \varphi_m$  for each  $m \in \mathbb{N}_0$ . Let  $\{P_k\} \subset P(H)$  be approximate unit of  $K(H)$ . Then by lemma 2.2 and linearity of  $\varphi_m$  we get

$$\begin{aligned} h_m(T) &= \lim_n \frac{1}{2^n} \varphi_m(2^n T) = \lim_{n,k} \frac{1}{2^n} \varphi_m(2^{n-1} T P_k + 2^{n-1} P_k T) \\ &= \lim_{n,k} \frac{1}{2^n} \sum_{l+j=m} [2^{n-1} \varphi_l(T) \varphi_j(P_k) + 2^{n-1} \varphi_j(T) \varphi_l(P_k)] \\ &= \lim_{n,k} \frac{1}{2} \sum_{l+j=m} [\varphi_l(T) \varphi_j(P_k) + \varphi_j(T) \varphi_l(P_k)] = \varphi_m(T) \end{aligned}$$

for all  $T \in K(H)$  and  $m \in \mathbb{N}_0$ . Now, Let  $S, T \in K(H)$ . There are compact self-adjoint operators  $S_1, S_2$  such that  $S = S_1 + iS_2 = \sum_{l=1}^{\infty} \alpha_l P_l + i \sum_{j=1}^{\infty} \beta_j P_j$  where  $P_k \in P(H)$  and  $\alpha_k, \beta_k \in \mathbb{C}$  for all  $k \in \{1, 2, 3, \dots\}$ . It follows from linearity and continuity of  $\varphi$  and  $T$  that

$$\begin{aligned}
\varphi_m(TS + ST) &= \varphi_m \left( T \left\{ \sum_{l=1}^{\infty} \alpha_l P_l + i \sum_{j=1}^{\infty} \beta_j P_j \right\} + \left\{ \sum_{l=1}^{\infty} \alpha_l P_l + i \sum_{j=1}^{\infty} \beta_j P_j \right\} T \right) \\
&= \sum_{l=1}^{\infty} \varphi_m(\alpha_l T P_l + \alpha_l P_l T) + i \sum_{j=1}^{\infty} \varphi_m(\beta_j T P_j + \beta_j P_j T) \\
&= \sum_{l=1}^{\infty} \sum_{s+k=m} [\varphi_s(T) \varphi_k(\alpha_l P_l) + \varphi_s(\alpha_l P_l) \varphi_k(T)] \\
&\quad + i \sum_{j=1}^{\infty} \sum_{s+k=m} [\varphi_s(T) \varphi_k(\beta_j P_j) + \varphi_s(\beta_j P_j) \varphi_k(T)] \\
&= \sum_{s+k=m} \varphi_s(T) \left\{ \sum_{l=1}^{\infty} \varphi_k(\alpha_l P_l) + i \sum_{j=1}^{\infty} \varphi_k(\beta_j P_j) \right\} + \left\{ \sum_{l=1}^{\infty} \varphi_s(\alpha_l P_l) + i \sum_{j=1}^{\infty} \varphi_s(\beta_j P_j) \right\} \varphi_k(T) \\
&= \sum_{s+k=m} \varphi_s(T) \varphi_k(S) + \sum_{s+k=m} \varphi_k(T) \varphi_s(S)
\end{aligned}$$

This means that  $\phi$  is a cauchy–Jensen type higher Jordan derivation.  $\square$

**Corollary 3** Let  $p \in (0, 1), \theta \in [0, \infty)$  be real numbers. Suppose that  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is an approximate–strong of continuous mappings from  $K(H)$  into  $K(H)$  with  $\varphi_0(0) = 0$  such that,  $\varphi_m(2^n TP + 2^n PT) = \sum_{l+j=m} [2^n \varphi_l(T) \varphi_j(P) + 2^n \varphi_j(P) \varphi_l(T)]$  for each  $m \in \mathbb{N}_0, T \in K(H)$  and  $P \in P(H)$ .

If

$$\|2\varphi_m\left(\frac{T+S}{2} + \lambda R\right) - \varphi_m(T) - \varphi_m(S) - 2\lambda\varphi_m(R)\| < \theta(\|T\|^p + \|S\|^p + \|R\|^p)$$

for all  $\lambda \in \{1, i\}$  and all  $T, S, R \in K(H)$  and for each  $m \in \mathbb{N}_0$ , then  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is a Cauchy–Jensen type higher derivation.

**Proof.** Setting  $\phi(T, S, R) := \theta(\|T\|^p + \|S\|^p + \|R\|^p)$  all  $T, S, R \in K(H)$ . Then by Theorem 3.1 we get the desired result.  $\square$

**Corollary 4** Let  $p_1, p_2, p_3$  and  $\theta$  be positive real numbers with  $p_1 + p_2 + p_3 < 1$  and let  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is an approximate-strong of continuous mappings from  $K(H)$  into  $K(H)$  with  $\varphi_0(0) = 0$  such that,  $\varphi_m(2^n TP + 2^n PT) = \sum_{l+j=m} [2^n \varphi_l(T) \varphi_j(P) + 2^n \varphi_j(P) \varphi_l(T)]$  for each  $m \in \mathbb{N}_0$ ,  $T \in K(H)$  and  $P \in P(H)$ .

If

$$\|2\varphi_m\left(\frac{T+S}{2} + \lambda R\right) - \varphi_m(T) - \varphi_m(S) - 2\lambda\varphi_m(R)\| < \theta(\|T\|_1^p \cdot \|S\|_2^p \cdot \|S\|_3^p)$$

for all  $\lambda \in \{1, i\}$  and all  $T, S, R \in K(H)$  and for each  $m \in \mathbb{N}_0$ , then  $\phi = \{\varphi_0, \varphi_1, \dots, \varphi_m, \dots\}$  is a Cauchy-Jensen type higher derivation.

**Proof.** Setting  $\phi(T, S, R) := \theta(\|T\|_1^p \cdot \|S\|_2^p \cdot \|R\|_3^p)$  all  $T, S, R \in K(H)$ . Then by Theorem 3.1 we get the desired result.  $\square$

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