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Hybrid model in network DEA

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Abstract

Traditional DEA models deal with measurements of relative efficiency of DMUs regarding multiple - inputs VS. multiple-outputs. One of the drawbacks of these model is the neglect of intermediate products or linkong activities. After pointing out needs for inclusion of them to DEA models. We propose hybrid model that can deal with intermediate products formally using this model we can evaluate divisional efficiency of decision making units (DMU) and we show this model with an example.

Keywords: DEA, Network DEA, Hybrid model, Divisional efficiency, Overal efficiency.

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1 Introduction

Data Envelopment Analysis are technique on base of math planning that evaluate relative efficiency of DMUs which are homological. Traditional DEA models deal with measurements of relative efficiency of DMUs regarding multiple-inputs VS multiple-outputs one of the drawbacks of these model is the neglect of internal or linking activities. This units called network, in this paper we offer model that we can evaluate efficiencies of mention units. For example (Fig. 1)

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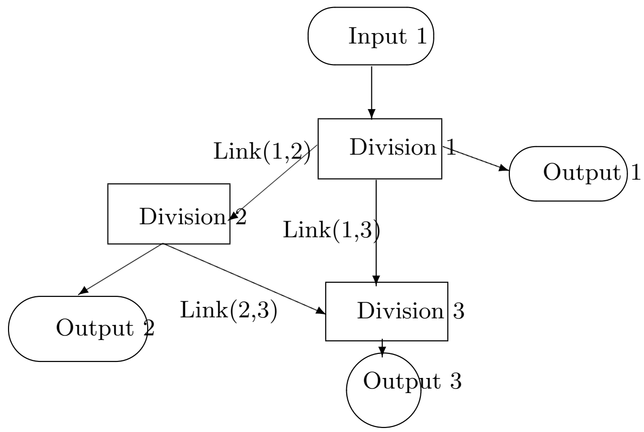


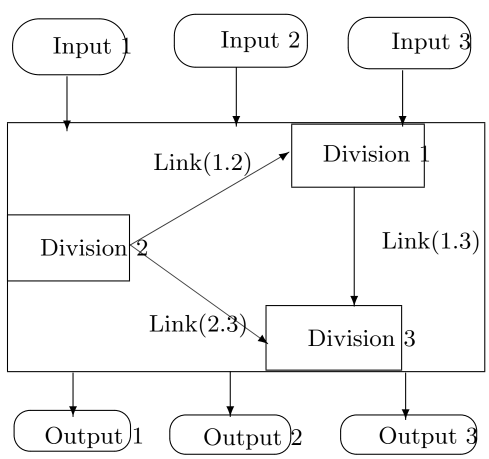
In the example the company has three division. Each division utilize its own input resources for producing its own outputs. However, there are linking activities (or intermediate products) as shown by Link 1-2, Link 1-3 and link 2-3, link 1-2 indicates that parts of the outputs from division 2.

In traditional DEA models, every activity should belong to either input or output but not to both. So usually they employ multiple steps for evaluation. Using intermediate products as outputs in one step and as input in other step. Thus, these models cannot deal with intermediate products directly in a single step.

Although there may be many variants of this process flow, the existence of linkong activities is an indispensable part of network DEA model within traditional DEA models there are at least two approaches of multi-division organization.

1.1 Aggregation (black box)

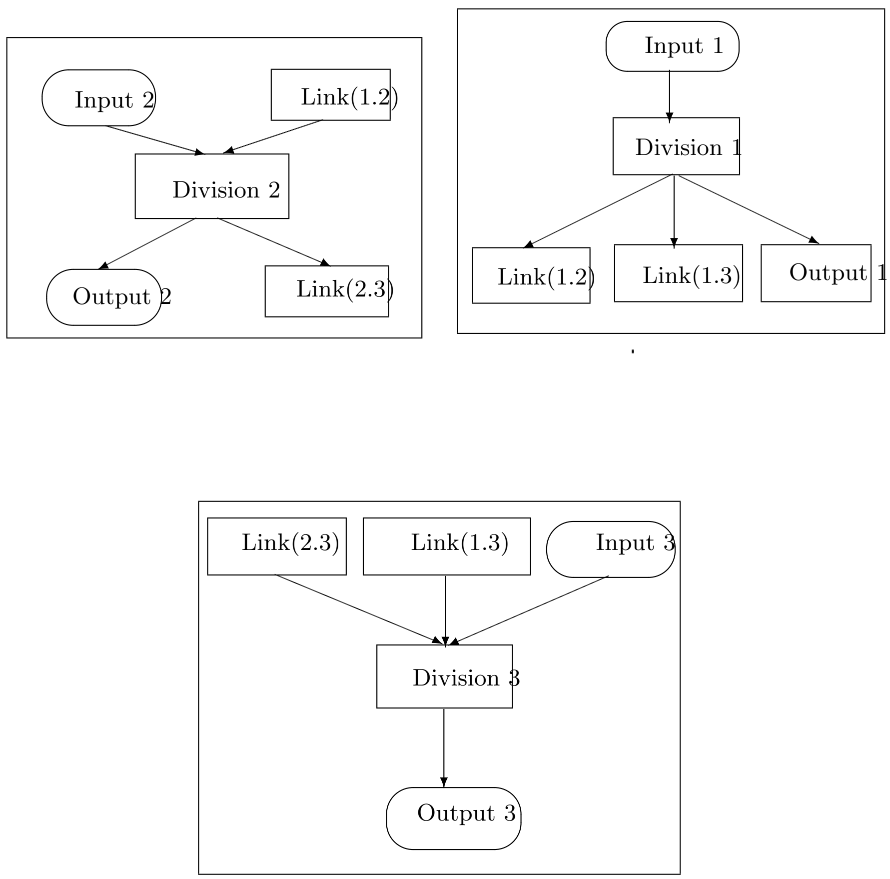
A simple approach is to aggregate these division into a single company which utilize inputs 1,2 and 3 and produces outputs 1.2 and 3 (Fig. 2).



However, using this approach we neglect internal linking and thus, we cannot evaluate the impact of division specific inefficiencies on the overall efficiency of company as a whole.

1.2 Separation

The second approach is to evaluate divisional efficiency individually (Fig. 3).



In this case, we value the efficiency of division 1 of each company among the set of DMUs using input 1 as input, and output 1, Link 1-2 and Link 1-3 as outputs. Similarly we evaluate efficiency of each division DMUs, and hence can find benchmarks for each division.

However this approach does not account for the continuity of links between divisions.

1.3 Needs for network DEA

We have alot of example in society such as hospital, power companies lead us to consider a DEA model called network DEA model (NDEA) that accounts for divisional efficiencies as well as the overall efficiency in a unified formwork. This means that we evaluate the total efficiency of DMUs as the main objective which involves divisional efficiencies as its components. for the first time DEA network were discussion by far and his co workers and were continued by other scientists and the result wea noticeable, extension of far s model were about radial models but many cases had many difficulties, for solving this problem tone and his co worker they presented network DEA a slack based measure approach model, we offer in this paper another model for evaluate efficiency called hybrid model.

2 Hybrid model in network DEA

In this section we introduce Hybrid model in network DEA, notation and production possibility set, efficiency.

2.1 Production possibility set

We deal with $n\ DMUs(j=1,\dots,n)$ consisting of k division $(k=1,\dots,K)$ let Mk and $Rk(k=1,\dots,K)$ be the numbers of inputs and outputs to division, respectively we denote the link leading from division k to division h by (k,h) and the set of link by L

$$L=\{(k,h)|h\in \text{ division } h, k\in \text{ division } k\}$$

The observed data are

$$x_{ij}^{k}\in R_{+}^{r_k},\quad j=1,\dots,n,k=1,\dots,k,i=1,\dots,k_k$$

(input resources to DMUj at division k) and

$$y_{r_j}^{k}\in R_{+}^{r_k},\quad j=1,\dots,n,k=1,\dots,k,r=1,\dots,r_k$$

(output products from DMUj at division k) and

$$z_{t_j}^{(k,h)}\in R_{+}^{\tau(k,h)},\quad j=1,\dots,h,(k,h)\in L,t=1,\dots,t_{(k,h)}$$

(linking intermediate products from division k to division h) where $t(k,h)$ is the number of items in link (k,h).

The production possibility set $(x^k,y^k,z^{(k,h)})$ is defined by

$$\begin{array}{ll}
x^k\geq\sum_j^n x_j^k\lambda_j^k & (k=1,\dots,m_k)\\
y^k\leq\sum_{j=1}^n y_j^k\lambda_j^k & (k=1,\dots,r_k)\\
z^{(k,h)}\leq\sum_{j=1}^n z_{ij}^{(k,h)}\lambda_j^k & (\forall(k,h))(\text{as outputs from k})\\
z^{(k,h)}\geq\sum_{j=1}^n z_{ij}^{(k,h)}\lambda_j^k & (\forall(k,h))(\text{as input to h})\\
\sum_{j=1}^n\lambda_j^k=1 & k=1,\dots,k\\
\lambda_j\geq 0, & j=1,\dots,n.
\end{array}$$

2.2 Efficiency

In this section, we introduce efficiency for hybrid model in network DEA we evaluate the input efficiency of DMUo by solving the following program

$$\theta_0^* \min \quad \frac{1-\sum_{k=1}^{k}(\frac{1}{m_k}(\sum_{i=1}^{m_k}(1-\theta_i^k)-\frac{1}{t_{(k,h)}}(\sum_{t=1}^{\tau_{(k,h)}}\frac{S_t^{k^-}}{Z_t^{t(k,h)}}))}{1+\sum_{k=1}^{K}(\frac{1}{r_k}(\sum_{i=1}^{r_k}(\phi_r^k-1)+\frac{1}{t_{(K,h)}}(\sum_{t=1}^{\tau_{(K,h)}}\frac{S_t^{k^+}}{Z_t^{t(k,h)}}))}$$

$$\text{s.t.} \quad \theta_t^k x_t^k \geq \sum_{j-1}^{n}\lambda_j^k x_{ij}^k \qquad k=1,\dots,K, i=1,\dots,m_k$$

$$\phi_r^k y_r^k \leq \sum_{j=1}^{n}\lambda_j^k y_{rj}^k \qquad k=1,\dots,K, r=1,\dots,r_k$$

$$\phi_r^k y_r^k \leq \sum_{j-1}^{n}\lambda_j^k y_{rj}^k \qquad k=1,\dots,K, r=1,\dots,r_k$$

$$z_t^{(k,h)} \leq \sum_{j=1}^{n} z_{t_j}^{(k,h)} l_j^k - s_t^{k^+}$$

$$z_t^{(k,h)} \leq \sum_{j=1}^{n} z_{t_j}^{(k,h)} l_j^k + s_t^k$$

$$\phi_r^k \geq 1 \qquad r=1,\dots,r_k, k=1,\dots,K$$

This model is fractional we change into linear then we show result of that model with an example.

Example. We present an illustrative example of electric power companies for describing Hybrid model in network DEA and compare the result traditional approaches.

For illustrative purpose, we choose ten vertically integrated power companies in the US. The inputs, outputs and links are as follows:
Generation (Div 1):
Input 1=labor input (number of employees)
Transmission (Div 2):
Input 2=labor input (number of employees)
Output 2=electric power sold to large customers
Link (1-2)=electric power generated
(Output from generation division and input to transmission division)
Link (2-3)= Electrict power sent
(Output from transmission division and to distribution division)

DMU	Div 1	Div 2		Div 3		Link	
	Input 1	Input 2	Output 1	Input 3	Output 3	Link 12	Link 23
A	0.838	0.27	0.879	0.962	0.337	0.894	0.362
B	1.233	0.132	0.538	0.0.443	0.18	0.678	0.188
C	0.321	0.045	0.911	0.482	0.198	0.836	0.207
D	1.483	0.111	0.57	0.467	0.491	0.869	0.516
E	1.592	0.208	1.086	1.073	0.372	0.693	0.407
F	0.79	0.139	0.722	0.545	0.253	0.966	0.269
G	0.451	0.075	0.509	0.366	0.241	0.647	0.257
H	0.408	0.74	0.619	0.229	0.097	0.756	0.103
I	1.864	0.061	1.023	0.691	0.38	1.191	0.402
J	1.222	0.149	0.769	0.337	0.178	0.792	0.187
Average	1.020	0.127	0.763	0.560	0.273	0.832	0.290

DMU	Overall Score		Separation			
		Overall Score	Overall Score	Divisional	Score	
			(NSBN)	Div 1	Div 2	Div 3
A	1.000	0.5984	0.659	0.633	0.662	0.684
B	0.531	0.8697	0.657	0.260	0.763	1.000
C	1.000	0.9838	0.984	1.000	1.000	0.959
D	1.000	0.8593	0.719	0.297	1.000	1.000
E	0.681	0.7186	0.844	1.000	0.635	0.792
F	1.000	0.6410	0.855	0.712	1.000	0.926
G	1.000	0.9703	0.893	0.787	0.890	1.000
H	1.000	0.9843	0.915	1.000	1.000	0.786
I	1.000	0.9843	0.915	1.000	1.000	0.786
J	1.000	0.9146	0.640	0.263	0.672	1.000
Average	0.921	0.8575	0.771	0.615	0.862	0.881
	Overall Score indicates $0.4 \times Div1 + 0.2 \times div2 + 0.4 \times Div3$					

Table 1 exhibits data for inputs, outputs, and link of the ten DMUs; A to J.

3 Concluding remarks

Consideration of the result show that evaluated efficiency by traditional DEA model with evaluated efficiency by hybrid model they are veri difference from each other, an evaluated efficiency by hybrid model is more accrete than traditional model.

References

[1] Network DEA: A slack - based measure approach by Kaoru Tone, Miki Tsutsui.

[2] Data Envelopment Analysis by Dr. Gr. Jahanshahloo, Dr. F. Hosseinzadeh Lotfi, Dr. H. Nikomaram.