Hybrid model in network DEA

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Abstract

Traditional DEA models deal with measurements of relative efficiency of DMUs regarding multiple-inputs VS multiple-outputs. One of the drawbacks of these models is the neglect of intermediate products or linking activities. After pointing out the need for inclusion of these activities to DEA models, we propose a hybrid model that can deal with intermediate products formally using this model. We can evaluate the efficiency of decision-making units (DMU) and show this model with an example.

Keywords: DEA, Network DEA, Hybrid model, Divisional efficiency, Overall efficiency.

1 Introduction

Data Envelopment Analysis are techniques on the base of math planning that evaluate relative efficiency of DMUs which are homological. Traditional DEA models deal with measurements of relative efficiency of DMUs regarding multiple-inputs VS multiple-outputs. One of the drawbacks of these models is the neglect of internal or linking activities. This units called network, in this paper we offer model that we can evaluate efficiencies of mention units. For example (Fig. 1)

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In the example the company has three divisions. Each division utilizes its own input resources for producing its own outputs. However, there are linking activities (or intermediate products) as shown by Link 1-2, Link 1-3 and Link 2-3, link 1-2 indicates that parts of the outputs from division 2.

In traditional DEA models, every activity should belong to either input or output but not both. So usually they employ multiple steps for evaluation. Using intermediate products as outputs in one step and as input in other step. Thus, these models cannot deal with intermediate products directly in a single step.

Although there may be many variants of this process flow, the existence of linking activities is an indispensable part of network DEA model within traditional DEA models there are at least two approaches of multi-division organization.

1.1 Aggregation (black box)

A simple approach is to aggregate these divisions into a single company which utilizes inputs 1, 2 and 3 and produces outputs 1, 2 and 3 (Fig. 2).
However, using this approach we neglect internal linking and thus, we cannot evaluate the impact of division specific inefficiencies on the overall efficiency of company as a whole.

1.2 Separation

The second approach is to evaluate divisional efficiency individually (Fig. 3).

In this case, we value the efficiency of division 1 of each company among the set of DMUs using input 1 as input, and output 1, Link 1-2 and Link 1-3 as outputs. Similarly we evaluate efficiency of each division DMUs, and hence can find benchmarks for each division.

However this approach does not account for the continuity of links between divisions.
1.3 Needs for network DEA

We have a lot of examples in society such as hospitals, power companies lead us to consider a DEA model called network DEA model (NDEA) that accounts for divisional efficiencies as well as the overall efficiency in a unified framework. This means that we evaluate the total efficiency of DMUs as the main objective which involves divisional efficiencies as its components. For the first time DEA network were discussed by Far and his co-workers and were continued by other scientists and the results were noticeable, extension of Far’s model were about radial models but many cases had many difficulties, for solving this problem Tone and his co-worker presented network DEA a slack based measure approach model, we offer in this paper another model for evaluate efficiency called hybrid model.

2 Hybrid model in network DEA

In this section we introduce Hybrid model in network DEA, notation and production possibility set, efficiency.

2.1 Production possibility set

We deal with \( n \) DMUs \((j = 1, \ldots, n)\) consisting of \( k \) division \((k = 1, \ldots, K)\) let \( M_k \) and \( R_k \) be the numbers of inputs and outputs to division, respectively we denote the link leading from division \( k \) to division \( h \) by \((k,h)\) and the set of link by \( L \)

\[
L = \{ (k,h) | h \in \text{division } h, k \in \text{division } k \}
\]

The observed data are

\[
x^{k}_{ij} \in R^r_k, \quad j = 1, \ldots, n, k = 1, \ldots, k, i = 1, \ldots, k
\]

(input resources to DMU \( j \) at division \( k \)) and

\[
y^{k}_{ir} \in R^r_k, \quad j = 1, \ldots, n, k = 1, \ldots, k, r = 1, \ldots, r
\]

(output products from DMU \( j \) at division \( k \)) and

\[
z^{(k,h)}_{ij} \in R^{(k,h)}, \quad j = 1, \ldots, h, (k,h) \in L, t = 1, \ldots, t_{(k,h)}
\]

(linking intermediate products from division \( k \) to division \( h \)) where \( t_{(k,h)} \) is the number of items in link \((k,h)\).

The production possibility set \((x^k, y^k, z^{(k,h)})\) is defined by

\[
x^k \geq \sum^n_j x^k_j \lambda^k_j
\]

\[
y^k \leq \sum^r_j y^k_j \lambda^k_j
\]

\[
z^{(k,h)} \leq \sum^n_j z^{(k,h)}_{ij} \lambda^k_j
\]

\[
z^{(k,h)} \geq \sum^n_j z^{(k,h)}_{ij} \lambda^k_j
\]

\[
\sum^k_j \lambda^k_j = 1 \quad k = 1, \ldots, k
\]

\[
\lambda^k_j \geq 0, \quad j = 1, \ldots, n.
\]
2.2 Efficiency

In this section, we introduce efficiency for hybrid model in network DEA. We evaluate the input efficiency of DMU0 by solving the following program:

\[
\theta^*_0 \min \frac{1 - \sum_{k=1}^K \left( \frac{1}{m_k} \left( \sum_{i=1}^{m_k} (1 - \theta_k^i) \right) - \frac{1}{\ell(k,h)} \left( \sum_{t=1}^{\gamma(k,h)} \frac{s^k_{t-} - s^k_{t+}}{Z_t^{(k,h)}} \right) \right)}{1 + \sum_{K=1}^K \left( \frac{1}{r_k} \left( \sum_{i=1}^{r_k} (\phi^k_r - 1) + \frac{1}{\ell(k,h)} \left( \sum_{t=1}^{\gamma(k,h)} \frac{s^k_{t+}}{Z_t^{(k,h)}} \right) \right) \right)}
\]

s.t. \[
\theta^k_i x^k_{ij} \geq \sum_{j=1}^n \lambda^k_j x^k_{ij} \\
\phi^k_r y^k_r \leq \sum_{j=1}^n \lambda^k_j y^k_{rj} \\
\phi^k_r y^k_r \leq \sum_{j=1}^n \lambda^k_j y^k_{rj} \\
z^{(k,h)}_t \leq \sum_{j=1}^n z^{(k,h)}_j t^k_j - s^k_+ \\
z^{(k,h)}_t \leq \sum_{j=1}^n z^{(k,h)}_j t^k_j + s^k_- \\
\phi^k_r \geq 1
\]

This model is fractional; we change it into linear then we show the results of that model with an example.

Example. We present an illustrative example of electric power companies for describing Hybrid model in network DEA and compare the result with traditional approaches.

For illustrative purpose, we choose ten vertically integrated power companies in the US. The inputs, outputs, and links are as follows:

Generation (Div 1):
- Input 1 = labor input (number of employees)

Transmission (Div 2):
- Input 2 = labor input (number of employees)
- Output 2 = electric power sold to large customers

Link (1-2) = electric power generated
(Output from generation division and input to transmission division)

Link (2-3) = Electric power sent
(Output from transmission division and to distribution division)
Table 1 exhibits data for inputs, outputs, and link of the ten DMUs; A to J.

3 Concluding remarks

Consideration of the result show that evaluated efficiency by traditional DEA model with evaluated efficiency by hybrid model they are veri difference from each other, an evaluated efficiency by hybrid model is more accrete than traditional model.

References


[2] Data Envelopment Analysis by Dr. Gr. Jahanshahloo, Dr. F. Hosseinzadeh Lotfi, Dr. H. Nikomaram.