



## Application of differential transformation method to the fisher equation.

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### Abstract

In this paper, the differential transform method (DTM) is applied to the Fisher equation. This method can be used to obtain the exact solutions of Fisher equation. Finally, we give some examples to illustrate the sufficiency of the method for solving such nonlinear partial differential equations. These results show that this technique is easy to apply.

*Key words:* Fisher equation, Differential, Transformation method, Spectral method.

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## 1 Introduction

In this paper, we consider the following Fisher equation

$$u_t - u_{xx} = \alpha u(1 - u). \quad (1.1)$$

The mathematical properties of Fisher's Equation have been studied extensively and there are numerous discussions in the literature. Recently, Wazwaz and Gorguis [23] studied the Fisher equation, the general Fisher equation, and nonlinear diffusion equation of the Fisher type subject to initial conditions by using Adomian decomposition method. A collocation method based on Whittakers sinc interpolation function [6] was also considered in [3]. A least-squares finite element method has been utilized in [9]. Authors of [11] considered nonlocal form of the Fisher equation. In [12], a Haar wavelet solution of Fishers equation has been presented. Homotopy perturbation method is applied to Fisher type equations by Agirseven and Ozis [2]. There are some other similar discussions. For example, authors of [10,18,17,19,21] presented several matrix formulation method for solving some equations with a boundary integral condition. The concept of the DTM was first proposed by Zhou [24], who solved linear and nonlinear problems in electrical circuit problems. Chen and Ho [7] developed this method for partial differential equations and Ayaz [4] applied it to the system of differential equations. A. Borhanifar and R. Abazari applied this method for Burgers and Schrödinger equations [1]. In [19,21,20,22,8,16], this method has been utilized for solving some important equations with initial and boundary conditions.

## 2 The Two-dimensional differential transform

The basic definitions and operations of one-dimensional DT are introduced in [24,7,4]. In order to speed up the convergence rate and improve the accuracy of calculation, the entire domain of  $t$  needs to be split into sub-domains [14,15].

Now we introduce the basic definition of the two-dimensional differen-

tial transform. To this end, consider a function of two variables  $w(x, t)$ , and suppose that it can be represented as a product of two single-variable functions, i.e.,  $w(x, t) = f(x)g(t)$ . Based on the properties of two-dimensional differential transform, the function  $w(x, t)$  can be represented as

$$w(x, t) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} W(i, j)x^i t^j. \quad (2.1)$$

where  $W(i, j)$  is called the spectrum of  $w(x, y)$ . Now we introduce the basic definitions and operations of two-dimensional DT as follows[10].

**Definition 1** *If  $w(x, t)$  is analytic and continuously differentiable with respect to time  $t$  in the domain of interest, then*

$$W(h, k) = \frac{1}{k!h!} \left[ \frac{\partial^{k+h}}{\partial x^k \partial t^h} w(x, t) \right]_{x=x_0, t=t_0}, \quad (2.2)$$

where the spectrum function  $W(k, h)$  is the transformed function, which is also called the T-function. Let  $w(x, y)$  be the original function while the uppercase  $W(k, h)$  stands for the transformed function. Now we define The differential inverse transform of  $W(k, h)$  as following:

$$w(x, t) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} W(k, h)(x - x_0)^k (t - t_0)^h. \quad (2.3)$$

Using Eq. (3) in (2), we have

$$w(x, t) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \frac{1}{k!h!} \left[ \frac{\partial^{k+h}}{\partial x^k \partial t^h} w(x, t) \right]_{x_0=0, t_0=0} x^k t^h = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} W(k, h)x^k t^h. \quad (2.4)$$

Now from the above definitions and Eqs. (2) and (3), we can obtain some of the fundamental mathematical operations performed by two-dimensional differential transform in Table 1.

**Table 1.** Operations of the two-dimensional differential transform

Original function	Transformed function
$w(x, t) = u(x, t) \pm v(x, t)$	$W(k, h) = U(k, h) \pm V(k, h)$
$w(x, t) = cu(x, t)$	$W(k, h) = cU(k, h)$
$w(x, t) = \frac{\partial}{\partial x} u(x, t)$	$W(k, h) = (k + 1)U(k + 1, h)$
$w(x, t) = \frac{\partial^{r+s}}{\partial x^r \partial t^s} u(x, t)$	$W(k, h) = \frac{(k+r)!(h+s)!}{k!h!} U(k+r, h+s)$
$w(x, t) = u(x, t)v(x, t)$	$W(k, h) = \sum_{r=0}^k \sum_{s=0}^h U(r, h-s)V(k-r, s)$
$w(x, t) = \frac{\partial}{\partial x} u(x, t) \frac{\partial}{\partial t} v(x, t)$	$W(h, k) = \sum_{r=0}^k \sum_{s=0}^h (k-r+1)(h-s+1) \times U(k-r+1, s)V(r, h-s+1)$

### 3 Application of the DTM

In this section, we apply the DTM for solving the presented Fisher equation.

**Remark 2** The symbol  $\otimes$  is used to denote the differential transform version of multiplication.

Then, Consider the following equation

$$u_t - u_{xx} = \alpha u(1 - u), \quad 0 \leq x \leq L, \quad t > 0, \quad (3.1)$$

with the initial condition

$$u(x, 0) = f(x), \quad 0 \leq x \leq L. \quad (3.2)$$

Let  $U(k, h)$  be the differential transform of  $u(x, t)$ . Applying Table 1., Eq. (2) and Definition 2.1 when  $x_0 = t_0 = 0$ , we get the differential transform version of Eq. (6) as follows:

$$(h + 1)U(k, h + 1) - \frac{(k+2)!}{k!}U(k + 2, h) = \alpha U(k, h) - \alpha u \otimes u \Big|_{x=k, t=h}. \quad (3.3)$$

By initial condition we get

$$\sum_{k=0}^{\infty} U(k, 0)x^k = \sum_{k=0}^{\infty} \frac{f^k(0)}{k!}x^k. \quad (3.4)$$

For  $k = 0, 1, 2, \dots$  the value of  $U(k, 0)$  can be obtained from Eq. (9). Using Eq. (8), we find the remainder values of  $U$ :

$$\begin{aligned} U(k, h+1) &= \frac{1}{h+1} \left( (k+1)(k+2)U(k+2, h) + \alpha U(k, h) \right. \\ &\quad \left. - \alpha \sum_{r=0}^k \sum_{s=0}^h U(r, h-s)U(k-r, s) \right), \\ k &= 0, 1, \dots, N, \quad h = 0, 1, \dots, N-1. \end{aligned} \quad (3.5)$$

**Example 3.1.** Consider the following Fisher equation [12]

$$u_t - u_{xx} = u(1-u),$$

with the initial condition

$$u(x, 0) = \lambda.$$

where  $\lambda$  is a constant.

From the above initial condition and Eq. (9), we have

$$U(0, 0) = \lambda, U(1, 0) = U(2, 0) = U(3, 0) = U(4, 0) = \dots = 0. \quad (3.6)$$

Using Eqs. (10) and (11), for  $h = 0$  and  $k = 0, 1, 2, \dots$ , we have

$$\begin{aligned} U(0, 1) &= \left( 2U(2, 0) + U(0, 0) - U(0, 0)U(0, 0) \right) = (\lambda - \lambda^2), \\ U(1, 1) &= \left( 6U(3, 0) + U(1, 0) - U(0, 0)U(1, 0) - U(1, 0)U(0, 0) \right) = 0, \\ U(2, 1) &= \left( 12U(4, 0) + U(2, 0) - U(0, 0)U(2, 0) \right. \\ &\quad \left. - U(1, 0)U(1, 0) - U(0, 0)U(2, 0) \right) = 0, \\ U(3, 1) &= 0, \\ U(4, 1) &= 0, \\ &\vdots, \end{aligned} \quad (3.7)$$

and for  $h = 1$  and  $k = 0, 1, 2, \dots$ , we get

$$\begin{aligned}
U(0, 2) &= \frac{1}{2} \left( 2U(2, 1) + U(0, 1) - U(0, 1)U(0, 1) - U(0, 1)U(0, 0) \right. \\
&\quad \left. - U(0, 0)U(0, 1) \right) = \frac{\lambda - \lambda^2}{2} + (-\lambda + \lambda^2)\lambda, \\
U(1, 2) &= \frac{1}{2} \left( 6U(3, 1) + U(1, 1) - U(0, 1)U(0, 1) - U(0, 0)U(1, 1) \right. \\
&\quad \left. - U(1, 1)U(0, 0) - U(1, 0)U(0, 1) \right) = 0 \\
U(2, 2) &= 0, \\
U(3, 2) &= 0, \\
&\vdots
\end{aligned} \tag{3.8}$$

By continuing this process, we obtain

$$u(x, t) \simeq \lambda + (\lambda - \lambda^2)t + \left( \frac{\lambda - \lambda^2}{2} + (-\lambda + \lambda^2)\lambda \right) t^2 + \dots, \tag{3.9}$$

which it is the Taylor expansion of the

$$u(x, t) = \frac{\lambda e^t}{1 - \lambda + \lambda e^t}, \tag{3.10}$$

which it is the exact solution of this example.

**Example 3.2.** Consider the Fisher equation [12]

$$u_t - u_{xx} = 6u(1 - u). \tag{3.11}$$

with the following initial condition

$$u(x, 0) = \frac{1}{(1 + e^x)^2}, \quad 0 \leq x \leq L,$$

From the above initial condition and Eq. (9), we have

$$\begin{aligned}
U(0,0) &= \frac{1}{2}, \\
U(1,0) &= -\frac{1}{4}, \\
U(2,0) &= 0, \\
U(3,0) &= \frac{1}{48}, \\
U(4,0) &= 0, \\
U(5,0) &= -\frac{1}{480}, \\
&\vdots
\end{aligned} \tag{3.12}$$

Using Eqs. (10) and (12), for  $h = 0$  and  $k = 0, 1, 2, \dots$ , we have

$$\begin{aligned}
U(0,1) &= \left( 2U(2,0) + 6U(0,0) - 6U(0,0)U(0,0) \right) = \frac{5}{4}, \\
U(1,1) &= \left( 6U(3,0) + 6U(1,0) - 6U(0,0)U(1,0) - 6U(1,0)U(0,0) \right) = 0, \\
U(2,1) &= \left( 12U(4,0) + 6U(2,0) - 6U(0,0)U(2,0) \right. \\
&\quad \left. - 6U(1,0)U(1,0) - 6U(0,0)U(2,0) \right) = -\frac{5}{16}, \\
U(3,1) &= 0, \\
U(4,1) &= \frac{5}{96}, \\
&\vdots,
\end{aligned} \tag{3.13}$$

and for  $h = 1$  and  $k = 0, 1, 2, \dots$ , we get

$$\begin{aligned}
U(0, 2) &= \frac{1}{2} \left( 2U(2, 1) + 6U(0, 1) - 6U(0, 1)U(0, 1) - 6U(0, 1)U(0, 0) \right. \\
&\quad \left. - 6U(0, 0)U(0, 1) \right) = 0, \\
U(1, 2) &= \frac{1}{2} \left( 6U(3, 1) + 6U(1, 1) - 6U(0, 1)U(0, 1) - 6U(0, 0)U(1, 1), \right. \\
&\quad \left. - 6U(1, 1)U(0, 0) - 6U(1, 0)U(0, 1) \right) = \frac{25}{16} \\
U(2, 2) &= 0, \\
U(3, 2) &= -\frac{25}{48}, \\
U(4, 2) &= 0, \\
U(2, 2) &= \frac{85}{768}, \\
&\vdots
\end{aligned} \tag{3.14}$$

If we continue this process, then we obtain the  $u(x, t)$  as follows

$$\begin{aligned}
u(x, t) &\simeq \left( \frac{1}{2} - \frac{1}{4} + \frac{1}{48} + \dots \right) \\
&+ \left( \frac{5}{4} - \frac{5}{16}x^2 + \frac{5}{96}x^4 + \dots \right)t \\
&+ \left( \frac{25}{16}x - \frac{25}{48}x^3 + \frac{85}{768}x^5 + \dots \right)t^2 + \dots,
\end{aligned} \tag{3.15}$$

which it is the Taylor expansion of the

$$u(x, t) = \frac{1}{(1 + e^{x-5t})^2} \quad 0 \leq x \leq L, \tag{3.16}$$

which it is the exact solution of the example 3.2.

## 4 Conclusions

In this research, the Differential Transformation method has been applied for finding exact solution of the Fisher equation with an initial condition.

By using this method, Numerical/analytical results obtained by a simple iterative process. The numerical results prove that this method is a powerful technique for nonlinear equations.

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