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Fixed Point Type Theorem In *S*-Metric Spaces (II)

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Abstract

In this paper, we prove some common fixed point results for two self mappings f and g on S-metric space such that f is a g.w.c.m with respect to g.

Key words: S-metric spaces, G.w.c.m, Generalized weakly contraction
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1 Introduction

During recent years, the fixed point theorems have become a main part of pure and applied sciences. Actually, it has become the basic tools in nonlinear functional analysis, optimization and economy. Gahler [4,5] introduced the notion of 2-metric spaces. Furthermore, Mustafa and Sims

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[11] introduced the notion of generalized metric space, and called it G-metric space. After then, many authors studied fixed and common fixed points in generalized metric spaces see ([1,2,12,13]). In [14], S. Sedghi, N. Shobe and A. Aliouche have introduced the notion of an S-metric space. Moreover, in [9,10] some new properties of S-metric spaces were represented. In present paper, we are going to prove some common fixed point theorems for two self-mappings f and g on S-metric space such that f is a g.w.c.m with respect to g.

2 Basic Concepts

First time the concept of S-metric spaces introduced by [14] as follows:

Definition 2.1 (See[14]). *Let X be nonempty set. An S-metric on X is a function $S : X^3 \to [0, \infty)$ which satisfies the following conditions, for each $x, y, z, a \in X$,*
(1) $S(x, y, z) \geq 0$,
(2) $S(x, y, z) = 0$ if and only if $x = y = z$,
(3) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$.
The pair (X, S) is called an S-metric space.

Example 2.2 *For any metric space (X, d),*
$S(x, y, z) = d(x, y) + d(x, z) + d(y, z)$ is an S-metric on X.

Example 2.3 *Let $\mathbb{R}$ be a real line. Then $S(x, y, z) = |x - z| + |y - z|$ for all $x, y, z \in \mathbb{R}$ is a S-metric on $\mathbb{R}$. This S-metric is called the usual S-metric on $\mathbb{R}$.*

Lemma 2.4 (See[14]) *In a S-metric space, we have $S(x, x, y) = S(y, y, x)$.*

There exists a natural topology on a S-metric spaces, for more details we refer to [9].

Lemma 2.5 (See[9]) *Any S-metric space is a Hausdorff space.*

Lemma 2.6 *Let (X, S) be a S-metric space. If there exist sequences*

$\{x_n\}$ *and* $\{y_n\}$ *such that* $\lim_{n\to\infty} x_n = x$ *and* $\lim_{n\to\infty} y_n = y$, *then* $\lim_{n\to\infty} S(x_n, x_n, y_n) = S(x, x, y)$.

Lemma 2.7 (See[14]). *Let* (X, S) *be an* S*-metric space. If the sequence* $\{x_n\}$ *in* X *converges to* x*, then* x *is unique.*

Definition 2.8 (See[6]). *A pair of maps f and g is called weakly compatible pair if they commute at coincidence points.*

Example 2.9 *Let* $X = [0, 3]$ *be equipped with the usual metric space* $d(x, y) =\mid x - y \mid$. *Define* $f, g : [0, 3] \to [0, 3]$ *by*

$$f(x) = \begin{cases} x & \text{if } x \in [0,1) \\ 3 & \text{if } x \in [1,3] \end{cases} \qquad g(x) = \begin{cases} 3 - x & \text{if } x \in [0,1) \\ 3 & \text{if } x \in [1,3] \end{cases}$$

Then for any $x \in [1, 3]$, $fg(x) = gf(x)$, *showing that* f, g *are weakly compatible maps on* $[0, 3]$.

Example 2.10 *Let* $X = \mathbb{R}$ *and define* $f, g : \mathbb{R} \to \mathbb{R}$ *by* $f(x) = \frac{x}{3}$, $x \in \mathbb{R}$ *and* $g(x) = x^2$,$x \in \mathbb{R}$. *Here* 0 *and* $\frac{1}{3}$ *are two coincidence points for the maps* f *and* g. *Note that* f *and* g *commute at* 0, *i.e.* $fg(0) = gf(0) = 0$, *but* $fg(\frac{1}{3}) = f(\frac{1}{9}) = \frac{1}{27}$ *and* $gf(\frac{1}{3}) = g(\frac{1}{9}) = \frac{1}{81}$ *and so* f *and* g *are not weakly compatible maps on* $\mathbb{R}$.

Choudhury [3] introduced the concept of weakly C-contractive mapping as follows:

Definition 2.11 *([3]). A mapping* $T : X \to X$ *where* (X, d) *is a metric space is said to be weakly* C*-contractive if for all* $x, y \in X$, *the following inequality holds:*

$$d(Tx, Ty) \leq \frac{1}{2}(d(x, Ty) + d(y, Tx)) - \phi(d(x, Ty), d(y, Tx))$$

where $\phi : [0, +\infty)^2 \to [0, +\infty)$ *is a continuous function such that* $\phi(x, y) = 0$ *if and only if* $x = y = 0$.

For more details on weakly C-contraction we refer the reader to [3,7,16]. Next part referral to definition of weakly S-contractive for mapping

$f : X \to X$, that was exploited from [15], but for S-metric spaces.

Definition 2.12 *Let (X, S) be a S-metric space. A mapping $f : X \to X$ is said to be weakly S-contractive type mapping if for all $x, y, z \in X$, the following inequality holds:*

$$\begin{aligned} S(fx, fy, fz) \leq & \frac{1}{4}(S(x, x, fy) + S(y, y, fz) + S(z, z, fx)) \\ & - \phi(S(x, x, fy), S(y, y, fz), S(z, z, fx)), \end{aligned}$$

where $\phi : [0, +\infty)^3 \to [0, +\infty)$ is a continuous function with $\phi(t, s, u) = 0$ if and only if $t = s = u = 0$.

Khan et al. [8] introduced the concept of altering distance function. Here, we attention to the following definition:

Definition 2.13 *The function $\psi : [0, +\infty) \to [0, +\infty)$ is called an altering distance function if the following properties are satisfied:*
(a1) ψ is continuous and increasing;
(a2) $\psi(t) = 0$ if and only if $t = 0$.

3 Main Result

Let (X, S) be an S-metric space and $f, g : X \to X$ be two mappings. We say that f is a generalized weakly contraction mapping (g.w.c.m) with respect to g if for all $x, y \in X$, the following inequality holds:

$$\begin{aligned} \psi(S(fx, fx, fy)) \leq & \psi\left(\frac{1}{4}(S(gx, gx, fx) + S(gx, gx, fy) + S(gy, gy, fx))\right) \\ & - \phi(S(gx, gx, fx), S(gx, gx, fy), S(gy, gy, fx)), \quad (3.1) \end{aligned}$$

where
(b1) ψ is an altering distance function;
(b2) $\phi : [0, +\infty)^3 \to [0, +\infty)$ is a continuous function with $\phi(t, s, u) = 0$ if and only if $t = s = u = 0$.

Theorem 3.1 *Let (X,S) be an S-metric space and $f,g : X \to X$ be two mappings such that f is a g.w.c.m with respect to g. Assume that*
(c1) $f(X) \subseteq g(X)$,
(c2) $g(X)$ is a complete subset of (X,S),
(c3) f and g are weakly compatible maps.
Then f and g have a unique common fixed point.

Proof. Since $f(X) \subseteq g(X)$, we can construct a sequence x_n in X such that $gx_{n+1} = fx_n$ for any $n \in \mathbb{N}$. If for some n, $gx_{n+1} = gx_n$, then $gx_n = fx_n$, that is, f and g have a common fixed point. Thus, we may assume that $gx_{n+1} \neq gx_n$ for any $n \in \mathbb{N}$. For $n \in \mathbb{N}$, then by (3.1) and (iii), we get

$$\begin{aligned}
\psi(S(gx_n, gx_n, gx_{n+1})) &= \psi(S(fx_{n-1}, fx_{n-1}, fx_n)) \\
&\leq \psi\Bigg(\tfrac{1}{4}(S(gx_{n-1}, gx_{n-1}, fx_{n-1}) \\
&\quad +S(gx_{n-1}, gx_{n-1}, fx_n) + S(gx_n, gx_n, fx_{n-1}))\Bigg) \\
&\quad -\phi(S(gx_{n-1}, gx_{n-1}, fx_{n-1}), S(gx_{n-1}, gx_{n-1}, fx_n) \\
&\quad , S(gx_n, gx_n, fx_{n-1})) \\
&= \psi\Bigg(\tfrac{1}{4}(S(gx_{n-1}, gx_{n-1}, gx_n) \\
&\quad +S(gx_{n-1}, gx_{n-1}, gx_{n+1}) + S(gx_n, gx_n, gx_n)) \\
&- \ \phi(S(gx_{n-1}, gx_{n-1}, gx_n), S(gx_{n-1}, gx_{n-1}, gx_{n+1}) \\
&\quad , S(gx_n, gx_n, gx_n)) \\
&\leq \psi\Bigg(\tfrac{1}{4}(S(gx_{n-1}, gx_{n-1}, gx_n) \\
&\quad +S(gx_{n-1}, gx_{n-1}, gx_{n+1}))\Bigg) \\
&\leq \psi\Bigg(\tfrac{1}{4}(3S(gx_{n-1}, gx_{n-1}, gx_n) + S(gx_{n+1} \\
&\quad , gx_{n+1}, gx_n))\Bigg).
\end{aligned} \tag{3.2}$$

Since ψ is increasing, by (3.2) and Lemma 2.4 , we have

$$\begin{aligned} S(gx_n, gx_n, gx_{n+1}) &\leq \frac{1}{4}(S(gx_{n-1}, gx_{n-1}, gx_n) + S(gx_{n-1}, gx_{n-1}, gx_{n+1})) \\ &\leq \frac{1}{4}(3S(gx_{n-1}, gx_{n-1}, gx_n) + S(gx_n, gx_n, gx_{n+1})) \end{aligned} \tag{3.3}$$

Then, we have $S(gx_n, gx_n, gx_{n+1}) \leq S(gx_{n-1}, gx_{n-1}, gx_n)$ for any $n \geq 1$. Therefore $\{S(gx_n, gx_n, gx_{n+1}), n \in \mathbb{N}\}$ is a non-increasing sequence. Hence there exists $r \geq 0$ such that

$$\lim_{n \to +\infty} S(gx_n, gx_n, gx_{n+1}) = r. \tag{3.4}$$

Letting $n \to +\infty$ in (3.3), we get

$$r \leq \frac{1}{4}r + \frac{1}{4}\lim_{n \to +\infty} S(gx_{n-1}, gx_{n-1}, gx_{n+1}) \leq \frac{3}{4}r + \frac{1}{4}r = r$$

which implies that

$$\lim_{n \to +\infty} S(gx_{n-1}, gx_{n-1}, gx_{n+1}) = 3r. \tag{3.5}$$

Again, from (3.2) we have

$$\begin{aligned} \psi(S(gx_n, gx_n, gx_{n+1})) \leq \psi&\left(\frac{1}{4}(S(gx_{n-1}, gx_{n-1}, gx_n) + S(gx_{n-1}, gx_{n-1}, gx_{n+1}))\right) \\ &- \phi(S(gx_{n-1}, gx_{n-1}, gx_n), S(gx_{n-1}, gx_{n-1}, gx_{n+1}), \\ &\quad S(gx_n, gx_n, gx_n)). \end{aligned}$$

Letting $n \to +\infty$ and using (3.4), (3.5) and the continuities of ψ and ϕ, we get

$$\psi(r) \leq \psi(r) - \phi(r, 3r, 0),$$

and hence $\phi(r, 3r, 0) = 0$. By a property of ϕ, we deduce that $r = 0$, that is,

$$\lim_{n \to +\infty} S(gx_n, gx_n, gx_{n+1}) = 0 \tag{3.6}$$

Now, we show that $\{gx_n\}$ is a Cauchy sequence. Suppose, $\{gx_n\}$ is not a Cauchy sequence, that is, $\lim_{m,n\to+\infty} S(gx_m, gx_m, gx_n) \neq 0$. Then, there exists $\epsilon > 0$ for which we can find two subsequences $\{gx_{m(i)}\}$ and $\{gx_{n(i)}\}$ of $\{x_n\}$ such that $n(i)$ is the smallest index for which

$$n(i) > m(i) > i, \qquad S(gx_{m(i)}, gx_{m(i)}, gx_{n(i)}) \geq \epsilon. \tag{3.7}$$

This means that

$$S(gx_{m(i)}, gx_{m(i)}, gx_{n(i)-1}) < \epsilon. \tag{3.8}$$

Now, from (3.7),(3.8) and (iii), we have that

$$\begin{aligned}
\epsilon &\leq S(gx_{m(i)}, gx_{m(i)}, gx_{n(i)}) \\
&\leq 2S(gx_{m(i)}, gx_{m(i)}, gx_{m(i)-1}) + S(gx_{n(i)}, gx_{n(i)}, gx_{m(i)-1}) \\
&\leq 2S(gx_{m(i)}, gx_{m(i)}, gx_{m(i)-1}) + 2S(gx_{n(i)}, gx_{n(i)}, gx_{n(i)-1}) + S(gx_{m(i)-1}, gx_{m(i)-1}, gx_{n(i)-1}) \\
&\leq 2S(gx_{m(i)}, gx_{m(i)}, gx_{m(i)-1}) + 2S(gx_{n(i)}, gx_{n(i)}, gx_{n(i)-1}) + 2S(gx_{m(i)-1}, gx_{m(i)-1}, gx_{m(i)}) \\
&\quad + S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{m(i)}) \\
&< 2S(gx_{m(i)}, gx_{m(i)}, gx_{m(i)-1}) + 2S(gx_{n(i)}, gx_{n(i)}, gx_{n(i)-1}) + 2S(gx_{m(i)-1}, gx_{m(i)-1}, gx_{m(i)}) + \epsilon
\end{aligned}$$

Letting $i \to +\infty$ in the top inequalities and using (3.6), we get that

$$\begin{aligned}
\lim_{n\to\infty} S(gx_{m(i)}, gx_{m(i)}, gx_{n(i)}) &= \lim_{n\to\infty} S(gx_{n(i)}, gx_{n(i)}, gx_{m(i)-1}) \\
&= \lim_{n\to\infty} S(gx_{m(i)-1}, gx_{m(i)-1}, gx_{n(i)-1}) \\
&= \epsilon
\end{aligned} \tag{3.9}$$

By (3.1), we have

$$\begin{aligned}
&\psi(S(gx_{n(i)}, gx_{n(i)}, gx_{m(i)})) = \psi(S(fx_{n(i)-1}, fx_{n(i)-1}, fx_{m(i)-1})) \\
&\leq \psi\Big(\frac{1}{4}(S(gx_{n(i)-1}, gx_{n(i)-1}, fx_{n(i)-1}) + S(gx_{n(i)-1}, gx_{n(i)-1}, fx_{m(i)-1}) + S(gx_{m(i)-1}, gx_{m(i)-1}, fx_{n(i)-1}))\Big) \\
&- \phi(S(gx_{n(i)-1}, gx_{n(i)-1}, fx_{n(i)-1}) + S(gx_{n(i)-1}, gx_{n(i)-1}, fx_{m(i)-1}) + S(gx_{m(i)-1}, gx_{m(i)-1}, fx_{n(i)-1})) \\
&= \psi\Big(\frac{1}{4}(S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{n(i)}) + S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{m(i)}) + S(gx_{m(i)-1}, gx_{m(i)-1}, gx_{n(i)}))\Big) \\
&- \phi(S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{n(i)}) + S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{m(i)}) + S(gx_{m(i)-1}, gx_{m(i)-1}, gx_{n(i)})) \\
&\leq \psi\Big(\frac{1}{4}(S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{n(i)}) + S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{m(i)}) + S(gx_{m(i)-1}, gx_{m(i)-1}, gx_{n(i)}))\Big)
\end{aligned} \tag{3.10}$$

since ψ is increasing and by (iii), we get

$$S(gx_{n(i)}, gx_{n(i)}, gx_{m(i)})$$
$$\leq \frac{1}{4}(S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{n(i)}) + S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{m(i)}) + S(gx_{m(i)-1}, gx_{m(i)-1}, gx$$
$$\leq \frac{1}{4}(S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{n(i)}) + 2S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{m(i)-1}) + S(gx_{m(i)}, gx_{m(i)}, gx$$
$$+ 2S(gx_{m(i)-1}, gx_{m(i)-1}, gx_{n(i)-1}) + S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{n(i)})$$

Letting $i \to +\infty$ in the top inequalities, and using (3.6) and (3.9), we get that

$$\lim_{n\to\infty} S(gx_{n(i)-1}, gx_{n(i)-1}, gx_{m(i)}) = 3\epsilon \tag{3.11}$$

Now, letting $i \to +\infty$ in (3.10) and using (3.6), (3.9), (3.11) and the continuities of ψ and ϕ, we have

$$\psi(\epsilon) \leq \psi\Big(\frac{1}{4}(0, 3\epsilon, \epsilon)\Big) + \phi(0, 3\epsilon, \epsilon)$$

Hence, we get $\phi(0, 3\epsilon, \epsilon) = 0$ and hence, by a property of ϕ, we deduce $\epsilon = 0$, a contradiction. Thus $\{gx_n\}$ is a Cauchy sequence in $g(X)$. Since$(g(X), S)$ is complete, then there exist $t, u \in X$ such that $\{gx_n\}$ converges to $t = gu$, that is,

$$\lim_{n\to\infty} S(gx_n, gx_n, gu) = 0. \tag{3.12}$$

By Lemma 2.6 we have

$$\lim_{n\to\infty} S(gx_n, gx_n, fu) = S(gu, gu, fu). \tag{3.13}$$

Let us show that $fu = t$. By (3.1), we get

$$\begin{aligned}\psi(S(gx_{n+1}, gx_{n+1}, fu) &= \psi(S(fx_n, fx_n, fu))\\
&\leq \psi\Big(\frac{1}{4}(S(gx_n, gx_n, fx_n) + S(gx_n, gx_n, fu) + (gu, gu, fx_n))\Big)\\
&- \phi(S(gx_n, gx_n, fx_n), S(gx_n, gx_n, fu), (gu, gu, fx_n))\\
&= \psi\Big(\frac{1}{4}(S(gx_n, gx_n, gx_{n+1}) + S(gx_n, gx_n, fu) + (gu, gu, gx_{n+1}))\Big)\\
&- \phi(S(gx_n, gx_n, gx_{n+1}), S(gx_n, gx_n, fu), (gu, gu, gx_{n+1}))\end{aligned}$$

Letting $n \to +\infty$ and using (3.6), (3.12),(3.13) and the continuities of ψ and ϕ and using the fact that ψ is increasing, we get

$$\psi(S(gu, gu, fu)) \leq \psi\Big(\frac{1}{4}(S(gu, gu, fu)\Big) - \phi\Big(0, S(gu, gu, fu), 0)\Big) \quad (3.14)$$

Therefore, $S(gu, gu, fu) = 0$ and hence $fu = gu = t$. Then, u is a coincidence point of f and g, and since the pair f, g is weakly compatible, we have $ft = gt$. Now we prove that $ft = gt = t$. By (3.1), we have

$$\begin{aligned}
\psi(S(gt, gt, gx_{n+1}) &= \psi(S(ft, ft, fx_n)) \\
&\leq \psi\Big(\frac{1}{4}(S(gt, gt, ft) + S(gt, gt, fx_n) + (gx_n, gx_n, ft))\Big) \\
&- \phi(S(gt, gt, ft), S(gt, gt, fx_n), (gx_n, gx_n, ft)) \\
&= \psi\Big(\frac{1}{4}(S(gt, gt, gt) + S(gt, gt, gx_{n+1}) + (gx_n, gx_n, gt))\Big) \\
&- \phi(S(gt, gt, gt), S(gt, gt, gx_{n+1}), (gx_n, gx_n, gt))
\end{aligned}$$

Letting $n \to +\infty$ and using the fact that ψ is increasing and (2.4), we get

$$\begin{aligned}
&\psi(S(gt, gt, gu) \\
&\leq \psi\Big(\frac{1}{4}(0 + S(gt, gt, fu) + (gu, gu, ft))\Big) - \phi(0, S(gt, gt, fu), (gu, gu, ft)) \\
&= \psi\Big(\frac{1}{4}(2S(gt, gt, gu))\Big) - \phi(0, S(gt, gt, gu), (gu, gu, gt)) \\
&\leq \psi\Big(S(gt, gt, gu)\Big) - \phi(0, S(gt, gt, gu), (gt, gt, gu))
\end{aligned}$$

which is true if $\phi(0, S(gt, gt, gu), S(gt, gt, gu)) = 0$, that is, $gt = gu = t$. We deduce that $t = gt = ft$, and so t is a common fixed point of f and g.

To prove the uniqueness, let v be another common fixed point of f and

g. By (3.1), we have

$$\begin{aligned}\psi(S(t,t,v)) &= \psi(S(ft,ft,fv)\\
&\psi\Big(\frac{1}{4}(S(ft,ft,ft)+S(ft,ft,fv)+S(fv,fv,ft))\Big)\\
&-\phi(S(ft,ft,ft),S(ft,ft,fv),S(fv,fv,ft))\\
&\leq \psi\Big(\frac{1}{4}(0+S(t,t,v)+S(v,v,t)\Big)-\phi(0,S(t,t,v),S(v,v,t))\\
&\leq \psi\Big(S(t,t,v)\Big)-\phi(0,S(t,t,v),S(t,t,v))\end{aligned}$$

Therefore, $\phi(0,S(t,t,v),S(t,t,v)) = 0$ and hence $S(t,t,v) = 0$. Thus $t = v$. □

Example 3.2 *Let $X = [0,2]$, and S be the usual S-metric on X. Moreover $\psi(t) = t/2$, $\phi(t,s,u) = \frac{t+s+u}{k}$ with $k \geq 8$, $fx = 1$ and $gx = 2-x$. It is easy to show that f is a g.w.c.m with respect to g. In fact, we have $\psi(S(fx,fx,fy)) = 0$,*

$$\psi\left(\frac{1}{4}(S(gx,gx,fx)+S(gx,gx,fy)+S(gy,gy,fx))\right) = \frac{1}{2}\left(\frac{1}{4}(4|1-x|+2|1-y|)\right)$$

and

$$\phi(S(gx,gx,fx),S(gx,gx,fy),S(gy,gy,fx)) = \frac{4|1-x|+2|1-y|)}{k}$$

Condition (3.1) *is trivially hold. Obviously, $f(X) \subseteq g(X)$, $g(X)$ is a complete subset of (X,S) and the pair $\{f,g\}$ is weakly compatible. Then, all the hypotheses of Theorem 3.1 are satisfied, and so f and g have a unique common fixed point, that is $x = 1$.*

Corollary 3.3 *Let (X,S) be a S-metric space and f , g be two self-mappings on X such that:*

$$S(fx,fx,fy) \leq \beta(S(gx,gx,fx)+S(gx,gx,fy)+S(gy,gy,fx)) \quad (3.15)$$

where $\beta \in [0,\frac{1}{4})$. Suppose that $g(X)$ is a complete subspace of (X,S), $f(X) \subseteq g(X)$ and the pair $\{f,g\}$ is weakly compatible. Then f and g have a unique common fixed point.

Proof. It's enough to put $\psi(t) = t$ and $\phi(t, s, u) = (\frac{1}{4} - \beta)(t + s + u)$ in Theorem 3.1. $\square$

Corollary 3.4 *Let (X, S) be a S-metric space and f , g be two self-mappings on X such that:*

$$\psi(S(fx, fx, fy)) \leq \psi\left(\frac{1}{4}(S(x, x, fx) + S(x, x, fy) + S(y, y, fx))\right) \\ - \phi(S(x, x, fx), S(x, x, fy), S(y, y, fx))$$

where $(b1)$ and $(b2)$ hold. Then f has a unique fixed point.

Proof. It suffices to put $g = Id_X$, the identity mapping on X in Theorem 3.1. $\square$

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