



# The tanh method for solutions of the nonlinear modified Korteweg de Vries equation

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## Abstract

In this paper, we have studied on the solutions of modified KdV equation and also on the stability of them. We use the tanh method for this investigation and given solutions are good-behavior. The solution is shock wave and can be used in the physical investigations

*Key words:* Solitons; Shock; Modified KdV equation; The tanh method

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## 1 Introduction

It is known that many phenomena in most of scientific fields can be described by nonlinear partial differential equations [2,3,4,11,12].

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Typical equations are the KdV equation, Burgers equation, Boussinesq equation and many others. Many problems, such as the KdV equation, involve dispersion, other problems, such as Burgers equation; involve dissipation, whereas other problems involve both dispersion and dissipation such as the BurgersKdV equation [3]. Typical problems of dispersion and dissipation are the flow of liquids containing gas bubbles and the propagation of waves on an elastic tube filled with a viscous fluid [3]. The explicit form of soliton solutions are derived using the Hirota's bilinear method in [8]. In recent years, many different Basis functions have been used to solve and reduce integral equations to a system of algebraic equations [1,9,10]. The tanh method is a powerful solution method for the computation of exact traveling wave solutions[13,14,15]. Various extension forms of the tanh method have been developed. First a power series in tanh was used as an ansatz to obtain analytical solutions of traveling wave type of certain nonlinear evolution equations. To avoid complexity, Malfliet [5,6,7] had customized the tanh technique by introducing tanh as a new variable, since all derivatives of a tanh are represented by a tanh itself. The next interest is in the determination of exact travelling wave solutions for generalized forms of Burgers equation, BurgersKdV equation, and BurgersHuxley equation. Searching for exact solutions of nonlinear problems has attracted a considerable amount of research work where computer symbolic systems facilitate the computational work. It is shown that infinite series which has been proposed for the solution can be cut into finite number (M) of series components. The parameter M, of the power series in tanh of the tanh method, plays a major role in this method in that it should be a positive integer to derive a closed form analytic solution. However, for non-integer values of M, we usually use a transformation formula to overcome this difficulty and to obtain exact travelling wave solutions. In what follows, the the tanh method will be reviewed briefly because details can be found in [5,6,7] and in [13,14,15,16,17,18]

## 2 Solution of modified KdV equation by the tanh method

We now consider the nonlinear modified KdV equation

$$u_t + Au u_{xx} + Bu_{xxx} - cu_{xx} = 0 \quad (2.1)$$

The wave variable  $\xi = x - vt$  carries (2.1) to

$$-vu_\xi + Au u_{\xi\xi} + Bu_{\xi\xi\xi} - cu_{\xi\xi} = 0 \quad (2.2)$$

The above equation is a differential equation with respect to  $\xi$ , and so by integrating we have

$$-vu + \frac{A}{2}u^2 + Bu'' - cu' = 0 \quad (2.3)$$

As stated before, the tanh method uses a finite series

$$u(x, t) = u(\mu\xi) = s(y) = \sum_{m=0}^M a_m y^m \quad (2.4)$$

where  $M$  is a positive integer, in most cases, that will be determined. However, if  $M$  is not an integer, a transformation formula is usually used. Substituting (2.4) into (2.3) yields an equation in powers of  $y$ . To determine the parameter  $M$ , we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms. With  $M$  determined, we collect all coefficients of powers of  $y$  in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters  $a_m$ , ( $m = 0 \dots, M$ ),  $\mu$  and  $\nu$ . Having determined these parameters, knowing that  $M$  is a positive integer in most cases, and using (2.4) we obtain an analytic solution in a closed form. We also introduce a new independent variable

$$y = \tanh(\mu\xi) \quad (2.5)$$

that leads to the change of derivatives

$$\frac{d}{d\xi} = \frac{d}{dy} \frac{dy}{d\xi} = \mu(1 - y^2) \frac{d}{dy} \quad (2.6)$$

$$\frac{d^2}{d\xi^2} = \frac{d}{d\xi} \frac{d}{d\xi} = \mu^2(1-y^2)\left(-2y\frac{d}{dy} + (1-y^2)\frac{d^2}{dy^2}\right)$$

where other derivatives can be derived in a similar manner. Therefore by replacing (2.4) in (2.3) and using (2.6), we derive an equation with the respect to  $u$  as follows

$$\begin{aligned} -v\left(\sum_{m=0}^M a_m y^m\right) + \frac{A}{2}\left(\sum_{m=0}^M a_m y^m\right)^2 \\ + B\mu^2(1-y^2)\left(-2y\frac{d}{dy}\left(\sum_{m=0}^M a_m y^m\right)\right) \\ + (1-y^2)\frac{d^2}{dy^2}\left(\sum_{m=0}^M a_m y^m\right) \\ - c\mu(1-y^2)\frac{d}{dy}\left(\sum_{m=0}^M a_m y^m\right) = 0. \end{aligned}$$

As mentioned above, to determine the parameter  $M$  we balance the linear terms of highest order in Eq. (2) with the highest order nonlinear term. This in turn gives

$$2M = 4 + M - 2 \quad (2.7)$$

so

$$M = 2 \quad (2.8)$$

This gives the solution in the form

$$s = \sum_{m=0}^M a_m y^m = a_0 + a_1 y + a_2 y^2 \quad (2.9)$$

Substituting (2.9) into (2), we will have

$$\begin{aligned} -v\left(a_0 + a_1 y + a_2 y^2\right) + \frac{A}{2}\left(a_0 + a_1 y + a_2 y^2\right)^2 \\ + B\mu^2\left(1-y^2\right)\left(-2y\left(a_1 + 2a_2 y\right) + 2a_2\left(1-y^2\right)\right) \\ - c\mu\left(1-y^2\right)\left(a_1 + 2a_2\right) = 0. \end{aligned}$$

or

$$\begin{aligned}
& -va_0 - va_1y - va_2y^2 + \frac{A}{2}a_0^2 + \frac{A}{2}a_1^2y^2 + \frac{A}{2}a_2^2y^4 + Aa_0a_1y + Aa_0a_2y^2 + \\
& Aa_1a_2y^3 - a_1c\mu - 2c\mu a_2y + ca_1\mu y^2 + 2c\mu a_2y^3 - 2Ba_1\mu^2y - 4a_2^B\mu^2 \\
& + 2Ba_2\mu^2 - 2a_1B\mu^2y^3 + 4a_2B\mu^2y^4 - 2a_2B\mu^2y^2 + 2a_2B\mu^2y^4 = 0.
\end{aligned}$$

Collecting the coefficients of different powers of variable  $y$  gives the system of algebraic equations for  $a_0$ ,  $a_1$  and  $a_2$

$$\begin{aligned}
y^0 : & \quad -va_1 - 2B\mu^2a_1 + Aa_0a_1 - 2\mu ca_2 = 0 \\
y^1 : & \quad -2va_2 - 16B\mu^2a_2 + 2Aa_0a_2 + Aa_1^2 + 2c\mu a_1 = 0 \\
y^2 : & \quad Aa_1a_2 + 2B\mu^2a_1 + 2c\mu a_2 = 0 \\
y^3 : & \quad Aa_2^2 + 12B\mu^2a_2 = 0
\end{aligned} \tag{2.10}$$

The mentioned system gives the following solutions for  $\mu$ ,  $a_0$ ,  $a_1$  and  $a_2$

$$\begin{aligned}
a_0 &= \frac{v}{A} + \frac{12B\mu^2}{A} \\
a_1 &= -\frac{12c\mu}{5A} \\
a_2 &= -\frac{12B\mu^2}{A} \\
\mu &= \frac{c}{10B}
\end{aligned} \tag{2.11}$$

Finally, the above coefficients give the following solution

$$u = \frac{v}{A} + \frac{3c^2}{25AB} - \frac{6c^2}{25AB} \tanh\left(\frac{c}{10B}\xi\right) - \frac{3c^2}{25AB} \tanh^2\left(\frac{c}{10B}\xi\right)$$

or

$$u(x, t) = u(\xi) = \frac{3c^2}{25AB} \left\{ 1 - \tanh^2\left(\frac{c}{10B}\xi\right) \right\} + \frac{v}{A} - \frac{6c^2}{25AB} \tanh\left(\frac{c}{10B}\xi\right)$$

The above result is a new solution for modified KdV equation which it is a shock wave. The transformation formula is used for every type of nonlinearity to show that our analysis is applicable to a variety of nonlinear problems.

### 3 Conclusion

The main aim of this work is to implement the standard tanh-function method and to emphasize its power. Our purpose is to overcome the situation where the parameter  $M$  is integer. It is well known that the parameter  $M$  will be normally a positive integer so that an analytic solution in a closed form can be derived. We have emphasized in this work that this relevant transformation is powerful and can be effectively used to discuss nonlinear evolution equations and related models in scientific fields. The goal of this work was achieved and travelling wave solutions were formally derived to generalized forms of KdV-Burgers equation. We derived a new formula for the KdV-Burgers equation where it has shock structure and it can be reduced to solitonic solution for well-known KdV equation. The change in physical structures of the obtained solutions needs physical explanation that is beyond the scope of this work.

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